College Of Pharmacy

T - Test

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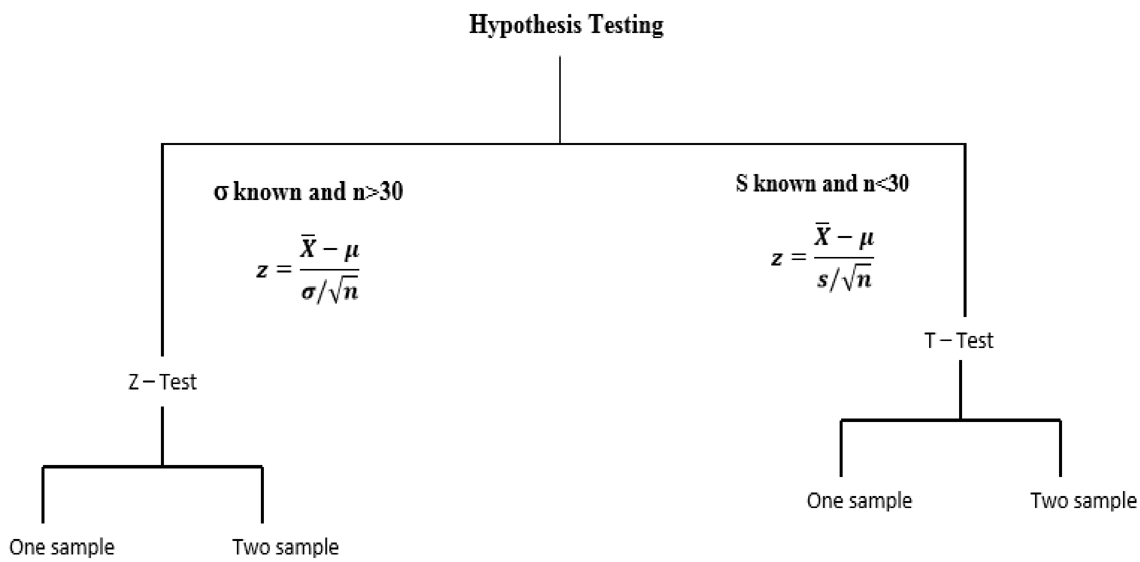
Chapter Four
Tests of Hypotheses on Population Means

There are two general methods used to make a **" good guess"** as to the true value of μ_0

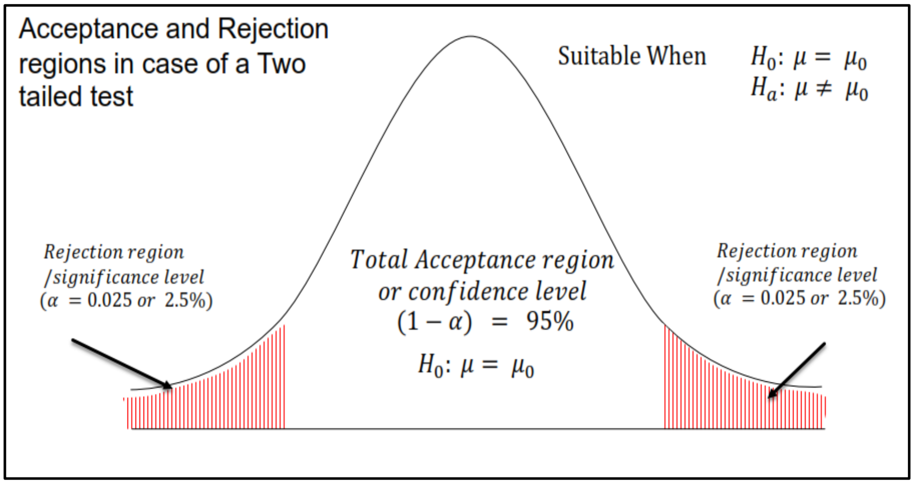
The first $\Rightarrow$involves determine a confidence interval on μ(CIμ).

The second $\Rightarrow$ concerned with making a guess as to the value of μ and then testing to see if such a guess is compatible with observed data . This is method is called hypothesis testing.

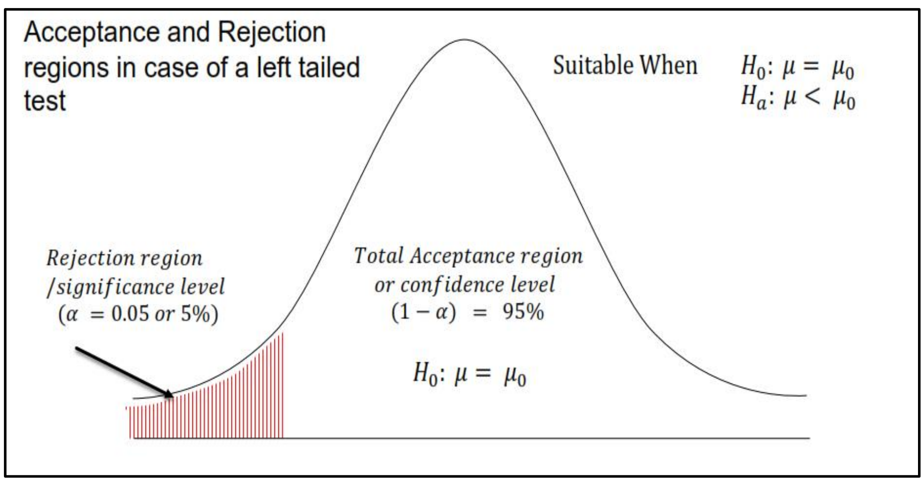


- Null Hypothesis is denoted by H_0 If a population mean is equal to hypothesized mean then Null Hypothesis can be written as H_0: $\mu = \mu_0$.
- The Alternative hypothesis is negation of null hypothesis and is denoted by H_1 If Null is given as $H_0 : \mu = \mu_0$ Then alternative Hypothesis can be written as $H_1 : \mu \neq \mu_0$
- Significance means the percentage risk to reject a null hypothesis when it is true and it is denoted by α generally taken as 1%, 5%, 10% , $(1 - \alpha)$ is the confidence interval in which the null hypothesis will exist when it is true.

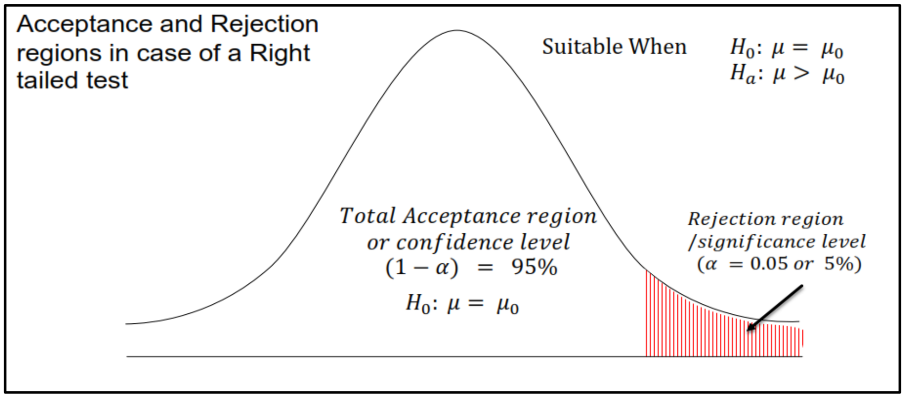
Two tailed test 5% Significance level



One left tailed test 5% Significance level



One Right tailed test 5% Significance level



- Decide a test statistics; z-test , t- test , F-test. Calculate the value of test statistics.
- Calculate the $T_{critical}$ at given Significance level from the table Compare the $T_{critical}$ value with $|T_{Statistic}|$ calculated value, If $T_{critical} > $ calculated value $|T_{statistical}|$ Accept H_0 or If $T_{critical} < |T_{statistical}|$ calculated value Rejected H_0

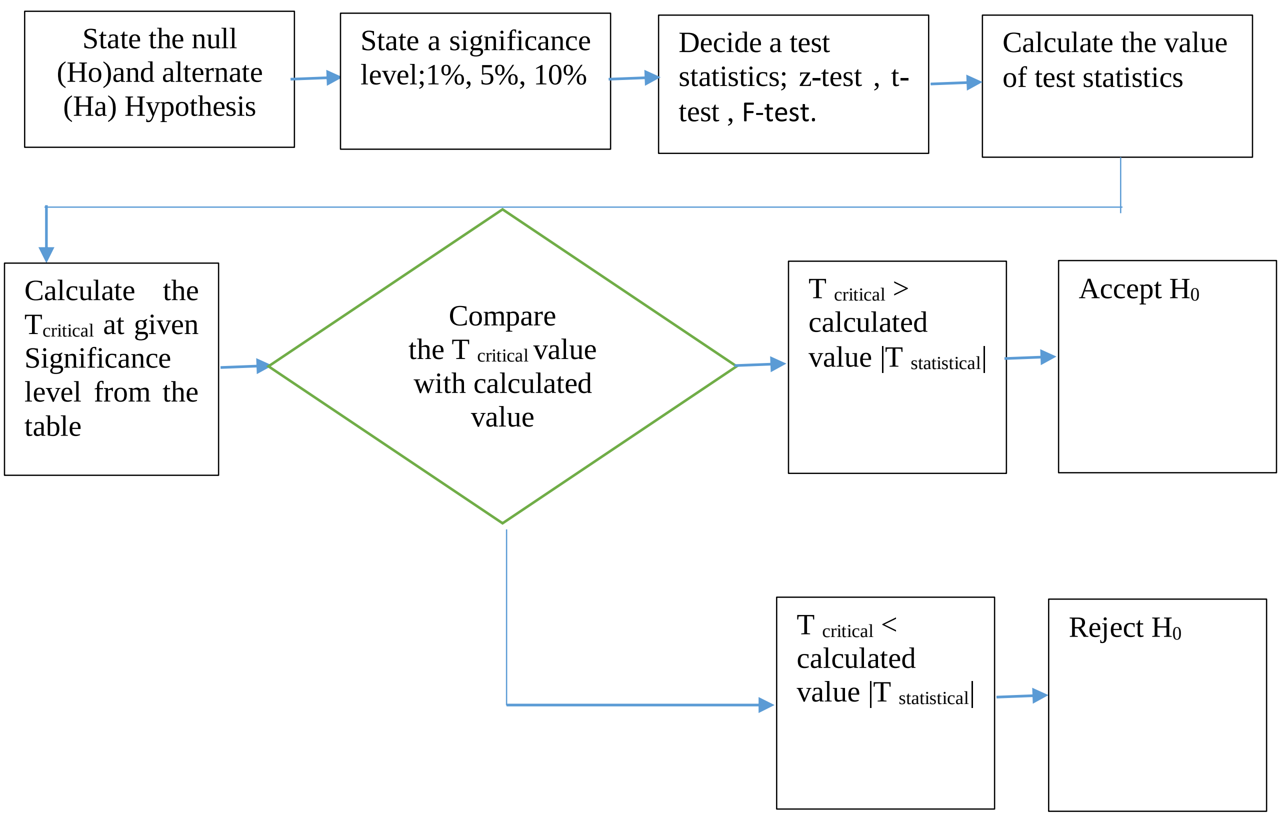


Table of $T_{Critical}$ values

Table of critical values of t:								
	One Tailed Significance level:							
	0.1	0.05	0.025	0.005	0.0025	0.0005	0.00025	0.00005
	Two Tailed Significance level:							
d.f.	0.2	0.1	0.05	0.01	0.005	0.001	0.0005	0.0001
2	1.89	2.92	4.3	9.92	14.09	31.6	44.7	100.14
3	1.64	2.35	3.18	5.84	7.45	12.92	16.33	28.01
4	1.53	2.13	2.78	4.6	5.6	8.61	10.31	15.53
5	1.48	2.02	2.57	4.03	4.77	6.87	7.98	11.18
6	1.44	1.94	2.45	3.71	4.32	5.96	6.79	9.08
7	1.41	1.89	2.36	3.5	4.03	5.41	6.08	7.89
8	1.4	1.86	2.31	3.36	3.83	5.04	5.62	7.12
9	1.38	1.83	2.26	3.25	3.69	4.78	5.29	6.59
10	1.37	1.81	2.23	3.17	3.58	4.59	5.05	6.21
11	1.36	1.8	2.2	3.11	3.5	4.44	4.86	5.92
12	1.36	1.78	2.18	3.05	3.43	4.32	4.72	5.7
13	1.35	1.77	2.16	3.01	3.37	4.22	4.6	5.51
14	1.35	1.76	2.14	2.98	3.33	4.14	4.5	5.36
15	1.34	1.75	2.13	2.95	3.29	4.07	4.42	5.24
16	1.34	1.75	2.12	2.92	3.25	4.01	4.35	5.13
17	1.33	1.74	2.11	2.9	3.22	3.97	4.29	5.04
18	1.33	1.73	2.1	2.88	3.2	3.92	4.23	4.97
19	1.33	1.73	2.09	2.86	3.17	3.88	4.19	4.9
20	1.33	1.72	2.09	2.85	3.15	3.85	4.15	4.84
21	1.32	1.72	2.08	2.83	3.14	3.82	4.11	4.78
22	1.32	1.72	2.07	2.82	3.12	3.79	4.08	4.74
23	1.32	1.71	2.07	2.81	3.1	3.77	4.05	4.69
24	1.32	1.71	2.06	2.8	3.09	3.75	4.02	4.65
25	1.32	1.71	2.06	2.79	3.08	3.73	4	4.62
26	1.31	1.71	2.06	2.78	3.07	3.71	3.97	4.59
27	1.31	1.7	2.05	2.77	3.06	3.69	3.95	4.56
28	1.31	1.7	2.05	2.76	3.05	3.67	3.93	4.53
29	1.31	1.7	2.05	2.76	3.04	3.66	3.92	4.51
30	1.31	1.7	2.04	2.75	3.03	3.65	3.9	4.48

Types of t-tests

A t-test is a hypothesis test of the mean of one or two normally distributed populations. Several types of t-tests exist for different situations, but they all use a test statistic that follows a t-distribution under the null hypothesis:

Test	Purpose	Example
1 sample t-test	Tests whether the mean of a single population is equal to a target value	Is the mean height of female college students greater than 5.5 feet?
2 sample t-test	Tests whether the difference between the means of two independent populations is equal to a target value	Does the mean height of female college students significantly differ from the mean height of male college students?
3-paired t-test	Tests whether the mean of the differences between dependent or paired observations is equal to a target value	If you measure the weight of male college students before and after each subject takes a weight-loss pill, is the mean weight loss significant enough to conclude that the pill works?

One Sample T-Test for testing means

Test Condition

- Population is infinite and normal,
- Sample size is small,
- Population variance is unknown
- H_1 may be one-sided or two sided

The hypotheses are:

Null hypothesis

H_0: $\mu = \mu_0$	The population mean (μ) equals the hypothesized mean (μ_0).

Alternative hypothesis

H_1: $\mu \neq \mu_0$	The population mean (μ) differs from the hypothesized mean (μ_0).

Test Statistics

$$\boldsymbol{t = \frac{x' - \mu}{\frac{s}{\sqrt{n}}}}$$

with *d.f* (degree of freedom) = n – 1

$$\text{where} \quad \boldsymbol{s = \sqrt{\frac{\sum(x_i - x')^2}{(n-1)}}}$$

In the formula that follows, we use a new symbol (μ) to indicate the population standard value, and s the standard deviation , x′= mean of the sample

Example

The following data represents hemoglobin values in gm/dl for 10 patients: 10.5.

10.5	9	6.5	8	11	7	7.5	8.5	9.5	12

Is the mean value for patients significantly differ from the mean value of general population (12 gm/dl). Evaluate the role of chance.**(α = 0.05)**

<u>Solution</u>

Mention all steps of testing hypothesis.

First, we must compute the mean (or average) of this sample:

$$x' = \frac{\sum x}{n} = \frac{10.5 + 9 + 6.5 + 8 + 11 + 7 + 7.5 + 8.5 + 9.5 + 12}{10} = 8.95$$

In the above example, there is some new mathematical notation.

x	x'	$(x - x')^2$
10.5	8.95	2.4025
9	8.95	0.0025

6.5	8.95	6.0025
8	8.95	0.9025
11	8.95	4.2025
7	8.95	3.8025
7.5	8.95	2.1025
8.5	8.95	0.2025
9.5	8.95	0.3025
12	8.95	9.3025
x′= 89.5/10		Σ***(x -x′)***^{2}**=** 722.4025

$$s = \sqrt{\frac{\sum(x_i - x')^2}{(n-1)}} = \sqrt{\frac{722.4025}{9}} = \mathbf{1.802005}$$

$$t = \frac{x' - \mu}{\frac{s}{\sqrt{n}}} = \frac{8.95 - 12}{\frac{1.802002}{\sqrt{10}}} = -5.35234$$

Then compare with tabulated value, for 9 df, and 5% level of significance. It is = 2.262, the calculated value>tabulated value. Reject Ho and conclude that there is a statistically significant difference between the mean of sample and population mean, and this difference is unlikely due to chance.

Hypothesis t- tests on the difference between to population means (μ_1 - μ_2)

Two types of t – tests for testing significance of difference between means will be present:

The pooled t- test and paired t – test. The distinction between these two lie in the method by which the samples are drawn.

The pooled t – test

The characteristic feature of the pooled is that the individual samples represent independent random samples from their respective populations. For example, in testing the effects of a new drug, an investigator may assign individuals at random to the treatment group and to the control group. Observations made on individual in the treatment group are independent of those made individuals in the control group .For pooled case, the number of individuals in the two samples need not be the same . The value for t is given by

$$t = \frac{(x'_1 - x'_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where S_p is called the pooled standard deviation , and is given by

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$t = \frac{(x'_1 - x'_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

d.f = $n_1 + n_2 - 2$

Example

The following data represents weight in Kg for 10 males and 12 females.

Male

80	75	95	55	60	70	75	72	80	65

Females:

60	70	50	85	45	60	80	65	70	62	77	82

2. Two independent samples, cont. Is there a statistically significant difference between, the mean weight of males and females. Let alpha = 0.01

To solve it follow the steps and use this equation :

$$t = \frac{(x'_1 - x'_2)}{\sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

	x	(x- x)^2		x′	(x- x′)^2
80	72.7	53.29	60	67.16667	51.36111
75	72.7	5.29	70	67.16667	8.027778
95	72.7	497.29	50	67.16667	294.6944
55	72.7	313.29	85	67.16667	318.0278

60	72.7	161.29	45	67.16667	491.3611
70	72.7	7.29	80	67.16667	164.6944
75	72.7	5.29	65	67.16667	4.694444
72	72.7	0.49	60	67.16667	51.36111
80	72.7	53.29	70	67.16667	8.027778
65	72.7	59.29	62	67.16667	26.69444
			77	67.16667	96.69444
			82	67.16667	220.0278
$Mean_1$=72.7		s^2 $=\frac{\sum(x-x')^2}{n-1}$	$Mean_2$= 67.16667		s^2 $=\frac{\sum(x-x')^2}{n-1}$
		128.4556			157.7879

$$t = \frac{(72.7 - 67.166)}{\sqrt{\frac{(12-1)157.7879 + (10-1)128.4556}{12+10-2}}\sqrt{\frac{1}{12}+\frac{1}{10}}}$$

t = 1.074

The tabulated t, 2 sides, for alpha 0.01 is 2.845 .Then accept Ho and conclude that there is no significant difference between the 2 means. This difference may be due to chance.

Note: - To calculated t- test using excel when α =0.05

t-Test: Two-Sample Independent

t-Test: Two-Sample Assuming Equal Variances		
	Variable 1	***Variable 2***
Mean	**72.7**	**67.16666667**
Variance	**128.4555556**	**157.7878788**
Observations	**10**	**12**
Pooled Variance	**144.5883333**	
Hypothesized Mean Difference	**0**	
df	**20**	
t Stat	**1.074730292**	
P(T<=t) one-tail	**0.147645482**	
t Critical one-tail	**1.724718243**	
P(T<=t) two-tail	**0.295290964**	
t Critical two-tail	**2.085963447**	

Decision:

We do a two-tail test. IF │t Stat │< t Critical, we accepted the null hypothesis. In the case 1.07473 < 2.0859. Therefore, we do accepted the null hypothesis; that means $\mu_0 = \mu_1$. We can make decision depended on the values of α and P $_{value}$ .IF α < P $_{value}$, we accepted the null hypothesis. In the case 0.05 < 0.2952. Therefore, we do accepted the null hypothesis; that means $\mu_0 = \mu_1$

The paired t – test

For the paired case, pairs are randomly selected from a single population. Each member of a pair is randomly assigned to one of the two treatments. The null hypothesis is that the mean different among pairs is zero. Example of pairing observation is the before and after measurements on the same individuals.

Formula:

$$\boldsymbol{t_{x\mathcal{D}} = \frac{\bar{D}}{SE_{diff}}}$$

t is the difference in means over a standard error

$$SE_{diff} = \frac{SD_D}{\sqrt{n_{pairs}}}$$

$$\text{where } SD_D = \sqrt{\frac{\sum d^2 - \frac{(\sum d)^2}{n}}{n-1}}$$

The standard error is found by finding the difference between each pair of observations. The standard deviation of these differences is SD_D. Divide SD_D by sqrt(number of pairs) to get SE_{diff}.

$$\boldsymbol{t_{x\mathcal{D}}} = \frac{\bar{\boldsymbol{D}}}{\frac{SD_D}{\sqrt{n_{pairs}}}}$$

where $d.f = n – 1$

Example Blood pressure of 8 patients, before & after treatment

Bp (before)	BP(after)	d	d^2
180	140	40	1600
200	145	55	3025
230	150	80	6400
240	155	85	7225
170	120	50	2500
190	130	60	3600
200	140	60	3600
165	130	35	1225
		$\Sigma d = 465$, $Mean_d = \frac{465}{8} = 58.123$	$\Sigma d^2 = 29175$

Results and conclusion

- t=9.387
- Tabulated t (df -7), with level of significance
- 0.05, two tails, = 2.36

We reject Ho and conclude that there is significant difference between BP readings before and after treatment.

- P<0.05.

Note: - To calculated t- test using excel when α =0.05

T-Test: Two-Sample dependent

t-Test: Paired Two Sample for Means		
	Variable 1	***Variable 2***
Mean	**196.875**	**138.75**
Variance	**720.9821429**	**133.9285714**
Observations	**8**	**8**
Pearson Correlation	**0.882107431**	
Hypothesized Mean Difference	**0**	
df	**7**	
t Stat	**9.387578897**	
P(T<=t) one-tail	**1.62001E-05**	
t Critical one-tail	**1.894578605**	
P(T<=t) two-tail	**3.24001E-05**	
t Critical two-tail	**2.364624252**	

Decision:

We do a two-tail test. IF │t Stat│ > t Critical, we rejected the null hypothesis. In the case 9.38757 < 2.3646. Therefore, we do rejected

the null hypothesis; that means $\mu_0 \neq \mu_1$. We can make decision depended on the values of α and P_{value} .IF $\alpha > P_{value}$, we rejected the null hypothesis. In the case 0.05 > 3.24001E-05. Therefore, we do rejected the null hypothesis; that means $\mu_0 \neq \mu_1$